ME 6402 – Lecture 28

PRACTICE PROBLEMS

April 22 2025

Overview:

- Kahoot Practice Problems
- Other Practice Problems

Kahoot Practice Problems

Kahoot Link

Other Practice Problems

1.

$$\dot{x}_1 = x_1^2 - x_1^3 + x_2$$
$$\dot{x}_2 = u$$

a) Apply backstepping to design a control law

b) Prove that the closed-loop system is asymptotically stable

2.

$$\dot{x}_1 = x_1^2 - x_1^3 + x_2$$
$$\dot{x}_2 = x_2 + 2u$$

a) Apply backstepping to design a control law

b) Prove that the closed-loop system is asymptotically stable

3.

$$\dot{x}_1 = x_1$$
$$\dot{x}_2 = x_2 + u$$
$$y = x_1$$

Does the system have a well-defined relative degree? If so, what is it?

4.

$$\begin{aligned}\dot{x}_1 &= -x_1 + \frac{2 + x_3^2}{1 + x_3^2} u \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_1 x_3 + u \\ y &= x_2\end{aligned}$$

Does the system have a well-defined relative degree? If so, what is it?

5.

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \varepsilon(1 - x_1^2)x_2 + u \\ y &= x_2\end{aligned}$$

Does the system have a well-defined relative degree? If so, what is it?

6.

$$\begin{aligned}\dot{x}_1 &= \cos(x_3)u_1 \\ \dot{x}_2 &= \sin(x_3)u_1 \\ \dot{x}_3 &= u_2 \\ y_1 &= x_1 \\ y_2 &= x_2\end{aligned}$$

Does the system have a well-defined vector relative degree? If so, what is it?

7.

$$\begin{aligned}\dot{x}_1 &= x_4 \cos(x_3) \\ \dot{x}_2 &= x_4 \sin(x_3) \\ \dot{x}_3 &= u_2 \\ \dot{x}_4 &= u_1 \\ y_1 &= x_1 \\ y_2 &= x_2\end{aligned}$$

Does the system have a well-defined vector relative degree? If so, what is it?

8.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &\sin(x_1) + u\end{aligned}$$

Is the system full-state feedback linearizable? If so, what is an output $y = h(x)$ that results in a full-state feedback linearized system?

9.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 + x_3^2 + u\\ \dot{x}_3 &= -x_3 + x_1\\ y &= x_1\end{aligned}$$

(a) Is the system input-output linearizable?

(b) If yes, transform it into the normal form and specify the region over which the transformation is valid.

10.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= -x_1^2 + u\end{aligned}$$

Synthesize a control barrier function to keep the state x_1 below a threshold of 2.